A New Multimodal Robust Optimization for Cogging Torque Reduction of Interior Permanent Magnet Motor

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The performance analysis of electromagnetic machines requires the computationally expensive finite element method because of the non-linear property of the electromagnetic field. Therefore, considering uncertainties such as manufacturing tolerances, a large amount of computation occurs in the design process. In this paper, the above robust design problem is solved by using a multimodal optimization. The multimodal optimization can provide low sensitivity to design variables as well a good fitness value without the Pareto front set. However, the multimodal optimization algorithm requires more computation compared to other problems because it has to find multiple peaks. To improve the convergence speed of the multimodal optimization, this paper developed an optimization algorithm based on surrogate model. The proposed algorithm can find solutions rapidly by estimating and correcting the peaks itself based on the surrogate model. In addition, the sensitivity to design parameters can be obtained directly from the first and second derivatives of the response surface without a sensitivity analysis. The developed algorithm is applied to several mathematical test functions, and its usefulness is verified by comparison with the results of conventional methods.

***Index Terms*—Electromagnetic machine design, multimodal optimization, robust optimization, surrogate model.**

I. Introduction

IN the electromagnetic machine (EM) design problem, there are many design variables to be determined, and their non-linear interrelations should be considered during the design process. To take the non-linear properties of electromagnetic field into account, a computationally expensive finite element analysis (FEA) is employed for EM design. Thus, an efficient optimization design strategy is required to reduce FEA evaluations and computation cost.

In real-world applications, the knowledge of multiple solutions is especially useful when the global optima may not always be feasible. In particular, physical constraints such as fabrication tolerance always exist in engineering problem. Consequently, the resultant performances can be different from intended ones. A multimodal optimization deals with such a problem by finding all or most of the multiple solutions that considers both fitness and solution quality. In recent years, several algorithms employing a niche concept have been studied for multimodal problems [1]-[2]. However, an excessive application of the niche concept drastically increases the objective function evaluations and the advantage from searching multiple minima can be lost.

In this paper, an optimization algorithm assisted by surrogate model has been newly developed for multimodal optimization problems. The surrogate model of the proposed algorithm reduces the iterations at early stage by providing approximate global and local minima locations. Moreover, the sensitivity for each design variable at optima can be predicted directly from the first and second order derivative to the surrogate model. Therefore, additional sensitivity analysis is not necessary. The developed multimodal optimization can reduce the computational cost significantly when compared to conventional algorithms. The validity of the proposed algorithm is proven through several mathematical test functions in the digest. The effectiveness of the proposed algorithm in electromagnetic machine design application will be presented in the full paper.

II. Proposed Optimization Algorithm

In this paper, we propose a new multimodal optimization strategy assisted by a surrogate model. There have been many interpolation techniques to generate a surrogate model such as the response surface method (RSM) and Kriging method. The Kriging method is modeled by a Gaussian process governed by a covariance between samplings and produces the best prediction of the intermediate values [3]. Hence, this paper employs the Kriging method. The proposed multimodal algorithm is summarized in Table I.

TABLE I
THE FLOW CHART OF PROPOSED ALGORITHM

Proposed multimodal optimization algorithm:
A. Construction of surrogate model
1. Definition of design constraints
Step 1. Define design variable and its constraints.
2. Construction of a surrogate model
Step 2. Generate initial samples
Generate initial samples in the design space based on the full factorial design (FFD) and calculate their objective function values.
Step 3. Construct a surrogate model:
Calculate a correlation matrix between the samples [3], and construct a surrogate model by applying the Kriging method.
Step 4. Predict objective function values at all feasible points based on the surrogate model and determine the global and local optima based on approximated first derivative of surrogate model:
The objective function values can be predicted by the surrogate model at all points. From the estimated objective values, a first-order derivative at an arbitrary point can be computed. The global and local peaks are found where the first-order derivative is switched from a minus to a plus in the minimizing problem, and vice versa in the maximizing problem.

Step 5. Update peaks and repeat step 3-5 until the number of solutions remains unchanged during 3 iterations:

The global and local solutions are updated at this step. When the present number of peaks is identical to that of the previous step during 3 iterations, the surrogate model is assumed to be converged. However, there are still errors between exact solutions and predicted peaks. Thus, to find exact solutions, an additional peak search process is necessary.

B. Searching for exact solutions

Step 6. Place additional samples around the predicted local peaks:

For an exact peak search, this paper focuses on target areas where peaks are expected to be. In this step, additional samples are scattered around the estimated peaks. The subpopulations around exact optima increase the response surface resolution, especially near the peaks. The high-resolution surrogate model can improve both peak prediction accuracy and sensitivity evaluation quality. The singular matrix problem caused by the subpopulation will be discussed in full paper.

Step 7. Update peaks and repeat step 6-7 until all of the solutions are no longer improved:

Based on the updated surrogate model, the new global and local peaks are obtained as shown in Fig 1 (a)-(c). When the newly developed solutions are no longer improved when compared to previous peaks, the process for searching exact solutions is terminated.

Step 8. Evaluate a sensitivity:

Based on the surrogate model, the sensitivity for 2-D problem is given by

$$S(X) = \sum_{i=1}^{n} \frac{f(x_i + dx_i, x_k) - 2f(X) + f(x_i - dx_i, x_k)}{dx_i^2} \qquad (1)$$

$$(k = 1, 2, \cdots, i-1, i+1, \cdots, n)$$

$$X = [x_1, \cdots, x_n] \qquad (2)$$

where f, dx_i, and n stand for the expected fitness value, difference of i^{th} design variable x_i, and number of design variables. When the sensitivity magnitude $S(X)$ has a low value, the convexity at the point is low and thus results in a flat response surface. The flat response surface indicates a low sensitivity to design variables, and is preferred in general design case. Based on the sensitivities and fitness values at each peak, a final solution is selected by a designer.

III. Numerical Tests and Results

The performance of the proposed algorithm is verified by comparison with the most famous multimodal algorithms; niching genetic algorithm (NGA) and auto tuning NGA. In NGA and auto-tuning NGA, the population size, crossover, and mutation probabilities are set to 20, 90% and 10%, respectively. As shown in Table II and III, the proposed method can achieve all solutions with much fewer fitness evaluations compared to conventional methods. The final solution is selected as (-4.56,-4.56) and (7.02, 6.96) for test function 1 and 2, respectively considering both fitness value and sensitivity to design variables.

IV. Conclusion

In the conventional robust optimization, a global solution is chosen from the Pareto front set consisting of fitness and sensitivity axes. The Pareto front set requires lots of fitness evaluations enough to get exact solutions. In this paper, by applying multimodal optimization which considers the sensitivity, a robust optimization is possible without the Pareto front set. Furthermore, this paper proposes an effective multimodal optimization algorithm that finds all of the solutions and their sensitivity with a remarkably small number of fitness value evaluations compared to the conventional multimodal optimization algorithms. Thus, the proposed method is especially useful in the design of electromagnetic machines that require a low number of fitness evaluations even with a non-linear response surface.

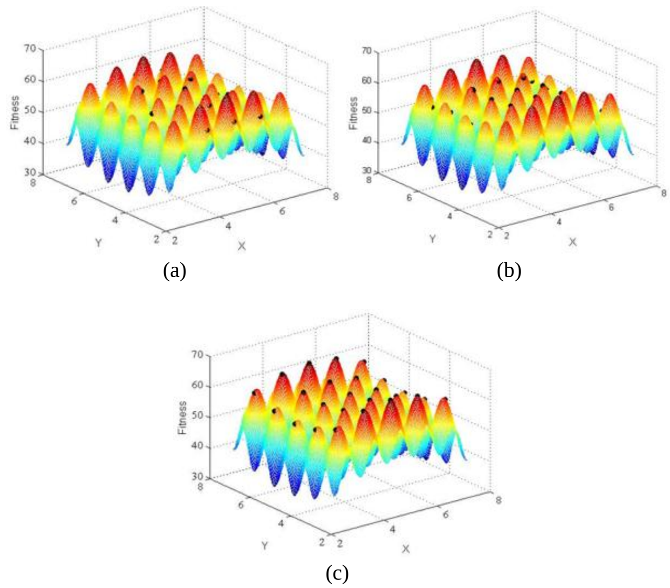

Fig. 1. Peak search process for the test function 2: (a) 86 samples, (b) 135 samples, and (c) 199 samples.

TABLE II

OPTIMIZATION RESULTS FOR TEST FUNCTION 1

$f(x, y) = 20+x^2 +y^2-10(\cos 2\pi x+ \cos 2\pi y)$, $-5.12 \leq x, y \leq 5.12$

	NGA	Auto tuning NGA	Proposed Algorithm
No. of peaks	100	100	100
No. of searched peaks	100	100	100
Fitness value evaluation	19,893	12,795	913

TABLE III

OPTIMIZATION RESULTS FOR TEST FUNCTION 2

$f(x, y) = (50-(x-5)^2+5\cos(2\pi(x-5))+(y-5)^2+5\cos(2\pi(y-5)))$, $2.5 \leq x, y \leq 7.5$

	NGA	Auto tuning NGA	Proposed Algorithm
No. of peaks	25	25	25
No. of searched peaks	25	25	25
Fitness value evaluation	7,890	4,961	241

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